

The influence of the Alfvénic drift on the shape of cosmic ray spectra in SNRs

V.N.Zirakashvili, V.S.Ptuskin

Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation, Russian Academy of Sciences, 142190 Troitsk, Moscow Region, Russia

Abstract Cosmic ray acceleration in SNRs in the presence of the Alfvénic drift is considered. It is shown that spectra of accelerated particles may be considerably softer in the presence of amplified magnetic fields.

Introduction

It is almost doubtless now that supernova remnants (SNRs) are the main source of galactic cosmic rays (CR). The outer shell of the exploding star moves with a supersonic velocity and produces a strong shock wave in the circumstellar medium. Diffusive shock acceleration [1, 2] results in the energy gain of energetic particles. The observation of high-energy TeV gamma-rays from several SNRs is the evidence of effective acceleration of cosmic ray particles up to the energy about 100 TeV [3] in these objects. This energy is only a factor of 30 lower than the "knee" energy $E_{knee} \sim 3$ PeV in the CR spectrum.

At present there exist two numerical models of nonlinear diffusive shock acceleration at the moving spherical shock of Berezhko and co-authors [4], and Jones and co-authors [5]. A so-called shock modification by the CR pressure is taken into account in these models. This is important since about 10 percent of supernova energy is transferred to accelerated CRs, if SNRs are the main source of CRs in the Galaxy. In addition it seems that CR particles are accelerated only at some part of the SNR shock, as it is observed in several SNRs.

These models may be used for modeling of CR acceleration in particular SNRs and for the calculation of an overall CR spectrum that is produced during lifetime of a SNR. It is expected that this spectrum is close to E^{-2} (see [6]). Slightly harder spectrum $E^{-1.9}$ was numerically obtained recently [7].

In this short report we present results of the new numerical model of CR acceleration in SNRs. It is shown, that CR spectra may be significantly softer if the advective velocity of CRs downstream of the shock is essentially different from the gas velocity. This situation is probable if the magnetic field is amplified in SNRs.

Model of nonlinear diffusive shock acceleration in SNRs.

Hydrodynamical equations for the gas density $\rho(r, t)$, gas velocity $u(r, t)$, gas pressure $P_g(r, t)$, and the equation for isotropic part of the CR momentum distribution $N(r, t, p)$ in the spherically symmetric case are given by

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u \rho \quad (1)$$

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - \frac{1}{\rho} \left(\frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \quad (2)$$

$$\frac{\partial P_g}{\partial t} = -u \frac{\partial P_g}{\partial r} - \frac{\gamma_g P_g}{r^2} \frac{\partial r^2 u}{\partial r} - (\gamma_g - 1)(w - u) \frac{\partial P_c}{\partial r} \quad (3)$$

$$\frac{\partial N}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 D(p, r, t) \frac{\partial N}{\partial r} - w \frac{\partial N}{\partial r} + \frac{\partial N}{\partial p} \frac{p}{3r^2} \frac{\partial r^2 w}{\partial r} + \frac{\eta \delta(p - p_{inj})}{4\pi p_{inj}^2 m} \rho(R + 0, t) (\dot{R} - u(R + 0, t)) \delta(r - R(t)) \quad (4)$$

Here $P_c = 4\pi \int p^2 dp v p N / 3$ is the CR pressure, $w(r, t)$ is the advective velocity of CRs, γ_g is the adiabatic index of the gas, and $D(r, t, p)$ is the CR diffusion coefficient. It was assumed that diffusive streaming of CRs results in the generation of magnetohydrodynamic (MHD) waves. CR particles are scattered by these waves. That is why the CR advective velocity w may differ from the gas velocity u . Damping of these waves results in additional gas heating. It is described by the last term in Eq. (3). The last term in Eq. (4) corresponds to the injection of thermal protons with momentum $p = p_{inj}$ and mass m at the shock front at $r = R(t)$. The dimensionless parameter η determines the injection efficiency.

CR diffusion is determined by magnetic inhomogeneities. Strong streaming of accelerated particles changes medium properties in the shock vicinity. CR streaming instability results in the high level of MHD turbulence [2] and even in the amplification of magnetic field in young SNRs [8]. Due to this effect the maximum energy of accelerated particles may be higher in comparison with previous estimates [9].

According to the recent numerical modeling of this instability, magnetic field is amplified by the flux of run-away highest energy particles in the relatively broad region upstream of the shock [10]. Magnetic energy density is a small fraction ($\sim 10^{-3}$) of the energy density of accelerated particles. This amplified

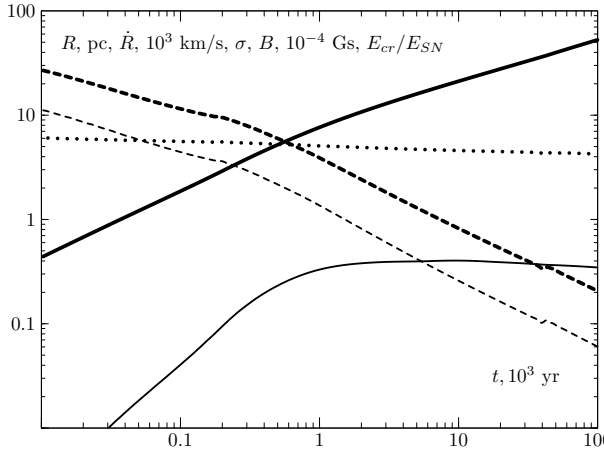


Fig.1. Dependencies on time of the shock radius R (thick solid line), shock velocity \dot{R} (thick dashed line), the total compression ratio of the shock σ (dotted line). Dependencies of the magnetic field strength downstream of the shock (dashed line) and ratio of CR energy and energy of supernova explosion E_{cr}/E_{SN} (solid line) are also shown.

almost isotropic magnetic field can be considered as a large-scale magnetic field for lower energy particles which are concentrated in the narrow region upstream of the shock. Streaming instability of these particles produces MHD waves propagating in the direction opposite to the CR gradient. This gradient is negative upstream of the shock and MHD waves propagate in the positive direction. The situation changes downstream of the shock where CR gradient is as a rule positive and MHD waves propagate in the negative direction. This effect is mostly pronounced downstream of the shock because the magnetic field is additionally amplified by the shock compression and the Alfvén velocity $V_A = B/\sqrt{4\pi\rho}$ may be comparable with the gas velocity in the shock frame $u' = \dot{R} - u(R=0, t)$. As for CR diffusion coefficient, it is probably close the Bohm value $D_B = pvc/3qB$, where q is the electric charge of particles.

Numerical modeling of CR acceleration in SNRs. We apply finite-difference method to solve Eqs (1-4) numerically upstream and downstream of the shock. The auto-model variable $\xi = r/R(t)$ is used instead of radius r . The non-uniform numerical grid upstream of the shock at $r > R$ allows to resolve small scales of hydrodynamical quantities appearing due to the pressure gradient of low-energy CRs. Eq. (4) for CRs was solved using an implicit finite-difference scheme. The explicit conservative TVD scheme [11] for hydrodynamical equations (1-3) was used. These equations are solved upstream the shock using the explicit finite-difference scheme.

We shall assume that the coordinate dependencies of the magnetic field and the gas density coincide:

$$B = \sqrt{4\pi\rho_0} \frac{\dot{R}\rho}{M_A\rho_0} \quad (5)$$

Here ρ_0 is the gas density of the circumstellar medium. The parameter M_A determines the value of the amplified magnetic field strength. The magnetic energy is about 3.5 percent of the dynamical pressure $\rho_0\dot{R}^2$, according to estimates from the width of X-ray filaments in young SNRs [12]. This number and characteristic compression ratio of a modified SNR shock $\sigma = 6$ correspond to $M_A \approx 23$.

CR advective velocity differs from the gas velocity on the value of the radial component of the Alfvén velocity V_{Ar} calculated in the isotropic random magnetic field: $w = u \pm V_{Ar}$. Here signs \pm correspond to regions upstream and downstream of the shock respectively. Using Eq. (5) we obtain

$$w = u \pm \frac{\dot{R}}{M_A} \sqrt{\frac{\rho}{3\rho_0}} \quad (6)$$

We shall use CR diffusion coefficient $D = D_B$ calculated with the magnetic field strength (5). Though in the real situation the level of MHD turbulence may drop with distance upstream of the shock and diffusion may be faster than the Bohm one (see [10]), we shall use these assumptions here (see also [7]).

The numerical results are obtained for the SNR shock propagating in the medium with the number density $n_0 = 0.1 \text{ cm}^{-3}$ and the temperature $T = 10^4 \text{ K}$. We use the ejecta mass $M_{ej} = 1.4M_\odot$, the energy of explosion $E_{SN} = 10^{51} \text{ erg}$ and the parameter of ejecta velocity distribution $k = 7$, corresponding to Ia supernovae. The initial shock velocity is $V_0 = 31000 \text{ km/s}$. The injection efficiency is taken in the form $\eta = 0.01\dot{R}/V_0$, and the injection momentum is $p_{inj} = 2m(\dot{R} - u(R=0, t))$. This dependence of the injection efficiency on the shock velocity results in the significant shock modification already at early stages of SNR expansion. This is in agreement with the observations of young extragalactic SNRs [13] and with the modeling of collisionless shocks [14].

The numerical results are obtained using the uniform grid with 800 cells downstream of the shock for the variable ξ , uniform grid with 800 cells upstream of the shock for the variable $\ln(\xi - 1 + 10^{-11})$. The uniform grid with 200 cells for the variable $\ln p/mc$ is used.

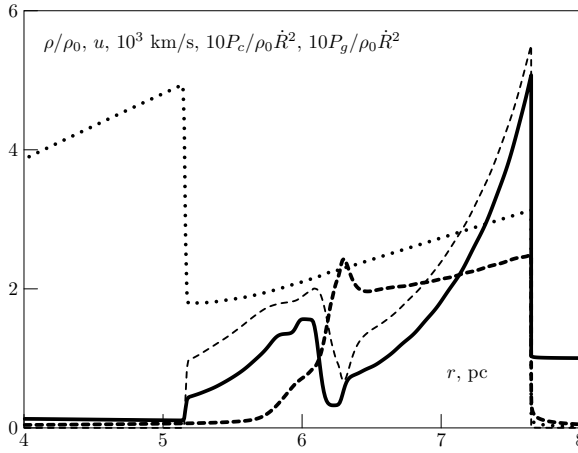


Fig.2. Radial dependencies of the gas density (thick solid line), the gas velocity (dotted line), CR pressure (thick dashed line), the gas pressure (dashed line) at $t = 10^3$ yr. At this moment of time the shock velocity is 3890 km/s, its radius is 7.6 pc, the magnetic field strength downstream of the shock is $136 \mu\text{G}$.

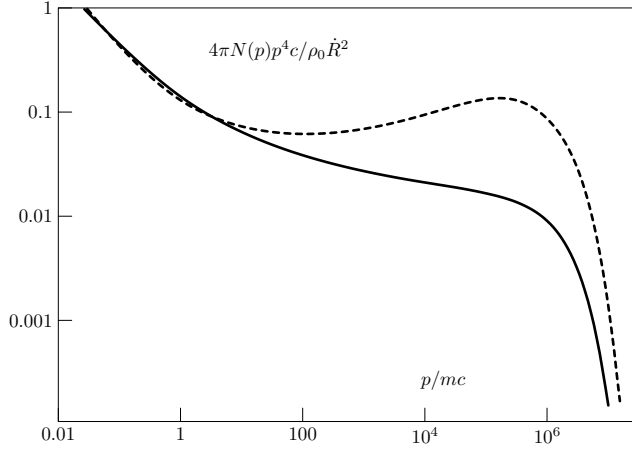


Fig.3. Spectrum of CR protons at the shock front at 10^3 yr after explosion. The results corresponding to the Eq. (6) (solid line) and to CR advection velocity coinciding with the gas velocity downstream of the shock (dashed line) are shown.

The dependencies on time of the shock radius R , the shock velocity \dot{R} , the total compression ratio of the shock σ , the magnetic field strength downstream of the shock and CR energy E_{cr}/E_{SN} are shown in Fig.1. The calculations are performed until the beginning of the radiative phase of SNR expansion at $t = 10^5$ yr, when the value of the shock velocity drops down to $\dot{R} = 206$ km/s. At this moment of time the maximum energy of particles accelerated in SNR is about 10 TeV, while higher energy particles have already leaved the remnant. This maximum energy may be significantly lower if one takes into account the wave damping on neutrals or nonlinear damping [15].

Radial dependencies of physical quantities at $t = 10^3$ yr are shown in Fig.2. The contact discontinuity between the ejecta and the interstellar gas is at $r = 6.2$ pc. The reverse shock in the ejecta is situated at $r = 5.2$ pc. We neglect the injection of thermal ions into diffusive shock acceleration at the reverse shock. At the Sedov stage the reverse shock moves in the negative direction and reach the center. The appearing reflected shock wave moves in the positive direction and overtakes the main shock wave after 40 thousand years after explosion. This results in the non-monotonic behavior of the shock velocity at this moment of time in Fig.1.

The spectrum of CR protons at the shock at $t = 10^3$ yr is shown in Fig.3 (solid line). It is compared with the proton spectrum obtained when the CR advection velocity coincides with the gas velocity downstream of the shock (dashed line). CR pressure is $P_c = 0.25\rho_0\dot{R}^2$ in the first case and a factor of two higher in the second one. The CR spectrum is significantly softer in the first case.

The spectrum of CR protons produced during the lifetime of the SNR $F(p)$ is shown in Fig.4 (solid line). It is determined by the integration of CR momentum distribution $N(p)$ on the simulation volume at the moment $t = 10^5$ yr. It is compared with the CR proton spectrum obtained when the CR advection velocity coincides with the gas velocity downstream of the shock (dashed line). The last curve is similar to results of Berezhko and Völk [7]. The integrated CR spectrum is again significantly softer in the first case.

Conclusion. Magnetic field amplification in SNRs may result in the significant difference of the CR advective velocity and the gas velocity downstream of the SNR shock. CR spectra in young SNRs and CR spectra produced by SNR during its lifetime may be significantly softer due to this effect. The fraction of supernova energy transferred to CRs is also reduced from 75 percent down to 35 percent of the mechanical energy of explosion. SNR luminosity in the TeV gamma-rays is reduced also. This work was supported by RFBR 07-02-00028 grant.

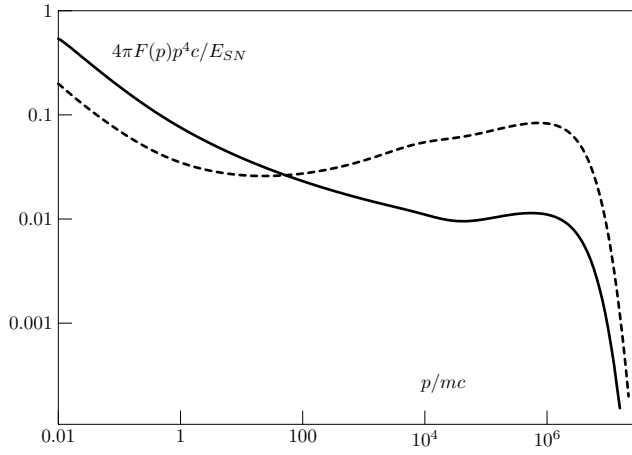


Fig.4. Spectrum of CR protons produced during the lifetime of SNR. The results corresponding to the Eq. (6) (solid line) and to CR advection velocity coinciding with the gas velocity downstream of the shock (dashed line) are shown.

References

- [1] Krymsky, G.F.//Sov. Phys.-Dokl. 1977. V.22. P.327
- [2] Bell, A.R., //Mon. Not. R. Astron. Soc. 1978, V. 182. P.147.
- [3] Aharonian F. et al.//Astron. Astrophys. 2007, V.464 P.253
- [4] Berezhko, E.G., Elshin, V.K., Ksenofontov, L.T.//Astropart. Phys. 1994. V.2. P.215
- [5] Kang, H., Jones, T.W.//Astropart. Phys. 2006. V.25 P.246
- [6] Ptuskin V.S., Zirakashvili V.N.//Astron. Astrophys., 2005.V.429.P.755
- [7] Berezhko E.G., Völk H.J.//Astrophys. J. 2007.V.661.P.L175
- [8] Bell, A.R., //Mon. Not. R. Astron. Soc. 2004, V.358. P.181.
- [9] Lagage, P.O., Cesarsky, C.J., //Astron. Astrophys. 1983. V.125, P.249.
- [10] Zirakashvili, V.N., Ptuskin, V.S.//Astrophys. J. 2008. V.678. P.939
- [11] Trac, H., Pen, U.//Publ. Astron. Soc. Pacific 2003, V.115. P.303
- [12] Völk H.J., Berezhko E.G., Ksenofontov L.T.//Astron. Astrophys. 2005.V.433.P.229
- [13] Chevalier, R.//Astrophys. J. 2006. V.651. P.381
- [14] Zirakashvili, V.N.//Astron. Astrophys. 2007. V.466. P.1
- [15] Ptuskin, V.S., Zirakashvili, V.N.//Astron. Astrophys. 2003. V.403. P.1